OCR
Oxford Cambridge and RSA

Wednesday 15 June 2016 – Morning

A2 GCE MATHEMATICS

4733/01 Probability & Statistics 2

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4733/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The results of 14 observations of a random variable V are summarised by

$$n = 14, \qquad \Sigma v = 3752, \qquad \Sigma v^2 = 1\,007\,448.$$

Calculate unbiased estimates of $\mathrm{E}(V)$ and $\mathrm{Var}(V)$. **[4]**

2 The mass, in kilograms, of a packet of flour is a normally distributed random variable with mean 1.03 and variance σ^2. Given that 5% of packets have mass less than 1.00 kg, find the percentage of packets with mass greater than 1.05 kg. **[6]**

3 The random variable F has the distribution $\mathrm{B}(40, 0.65)$. Use a suitable approximation to find $\mathrm{P}(F \leqslant 30)$, justifying your approximation. **[7]**

4 It is given that $Y \sim \mathrm{Po}(\lambda)$, where $\lambda \neq 0$, and that $\mathrm{P}(Y = 4) = \mathrm{P}(Y = 5)$. Write down an equation for λ. Hence find the value of λ and the corresponding value of $\mathrm{P}(Y = 5)$. **[5]**

5 55% of the pupils in a large school are girls. A member of the student council claims that the probability that a girl rather than a boy becomes Head Student is greater than 0.55. As evidence for his claim he says that 6 of the last 8 Head Students have been girls.

(i) Use an exact binomial distribution to test the claim at the 10% significance level. **[7]**

(ii) A statistics teacher says that considering only the last 8 Head Students may not be satisfactory. Explain what needs to be assumed about the data for the test to be valid. **[1]**

6 The number of cars passing a point on a single-track one-way road during a one-minute period is denoted by X. Cars pass the point at random intervals and the expected value of X is denoted by λ.

(i) State, in the context of the question, two conditions needed for X to be well modelled by a Poisson distribution. **[2]**

(ii) At a quiet time of the day, $\lambda = 6.50$. Assuming that a Poisson distribution is valid, calculate $\mathrm{P}(4 \leqslant X < 8)$. **[3]**

(iii) At a busy time of the day, $\lambda = 30$.

(a) Assuming that a Poisson distribution is valid, use a suitable approximation to find $\mathrm{P}(X > 35)$. Justify your approximation. **[6]**

(b) Give a reason why a Poisson distribution might not be valid in this context when $\lambda = 30$. **[1]**

7 A continuous random variable X has probability density function

$$f(x) = \begin{cases} ax^{-3} + bx^{-4} & x \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

(i) Explain what the letter x represents. **[1]**

It is given that $P(X > 2) = \frac{3}{16}$.

(ii) Show that $a = 1$, and find the value of b. **[7]**

(iii) Find $E(X)$. **[3]**

8 It is known that the lifetime of a certain species of animal in the wild has mean 13.3 years. A zoologist reads a study of 50 randomly chosen animals of this species that have been kept in zoos. According to the study, for these 50 animals the sample mean lifetime is 12.48 years and the population variance is 12.25 years^2.

(i) Test at the 5% significance level whether these results provide evidence that animals of this species that have been kept in zoos have a shorter expected lifetime than those in the wild. **[7]**

(ii) Subsequently the zoologist discovered that there had been a mistake in the study. The quoted variance of 12.25 years^2 was in fact the sample variance. Determine whether this makes a difference to the conclusion of the test. **[5]**

(iii) Explain whether the Central Limit Theorem is needed in these tests. **[1]**

9 The random variable R has the distribution $\text{Po}(\lambda)$. A significance test is carried out at the 1% level of the null hypothesis $H_0: \lambda = 11$ against $H_1: \lambda > 11$, based on a single observation of R. Given that in fact the value of λ is 14, find the probability that the result of the test is incorrect, and give the technical name for such an incorrect outcome. You should show the values of any relevant probabilities. **[6]**

END OF QUESTION PAPER